*Maths Challenge*

Alston

# Creative and Critical Thinking Skills

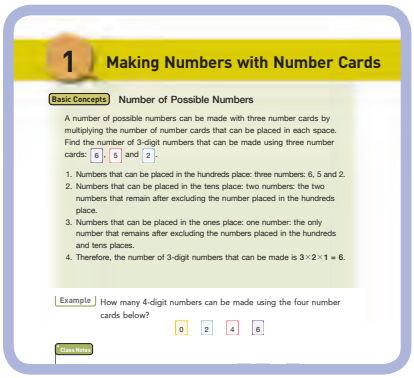
Ages 11-12
Intermediate




5B

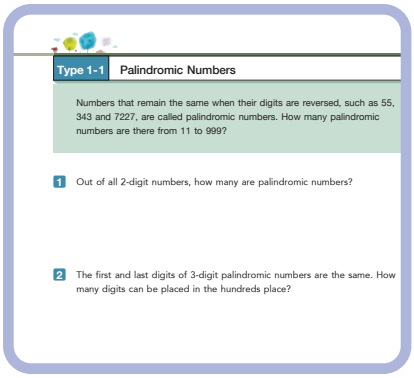
SAMPLE

# How to Use This Book



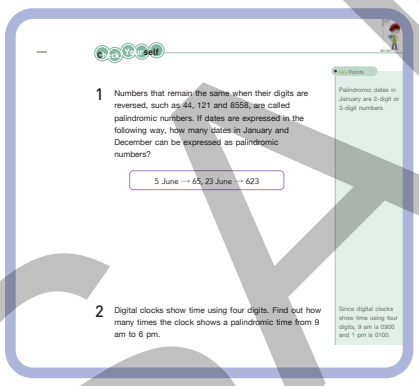

## Basic Concepts

**Creative Mathematics** begins right here! At the beginning of each lesson, **Basic Concepts** presents the main idea, along with various examples and diagrams that illustrate the main idea. The problems here are based on the basic concepts. The Class Notes guide you through the procedure step by step.



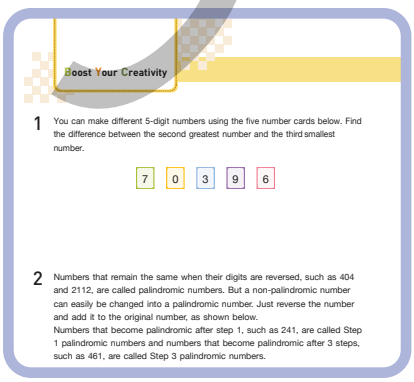

## Types

The main types of questions for each topic are introduced here. Follow the steps to find the solution to each problem.




## Check Yourself

Now that you have a good grasp of the basic concepts and types of problems, you can start on the different problems on your own. Key Points are provided to help you work towards the solution.




## Boost Your Creativity

This is where you practise applying your creativity to problems. Think creatively in order to reach a logical solution for each problem.

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# 1 Making Numbers with Number Cards

**Basic Concepts** Number of Possible Numbers

A number of possible numbers can be made with three number cards by multiplying the number of number cards that can be placed in each space. Find the number of 3-digit numbers that can be made using three number cards: 6, 5 and 2.

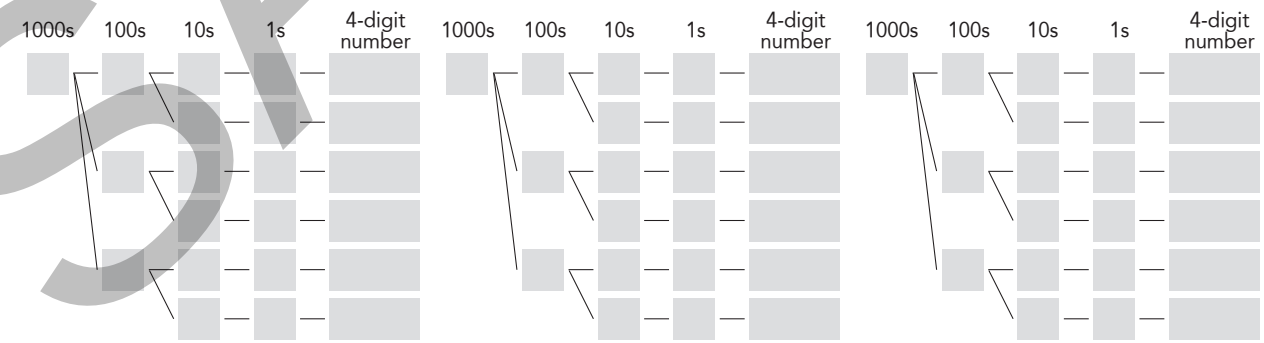
1. Numbers that can be placed in the hundreds place: three numbers: 6, 5 and 2.
2. Numbers that can be placed in the tens place: two numbers: the two numbers that remain after excluding the number placed in the hundreds place.
3. Numbers that can be placed in the ones place: one number: the only number that remains after excluding the numbers placed in the hundreds and tens places.
4. Therefore, the number of 3-digit numbers that can be made is $3 \times 2 \times 1 = 6$.

**Example** How many 4-digit numbers can be made using the four number cards below?

0 2 4 6

**Class Notes**

1. Place these three numbers in the thousands place: ☐, ☐ and ☐, and draw a tree diagram of all the possible 4-digit numbers.




2. When written as a multiplication problem, the equation looks like this:

1000s 100s 10s 1s

3 × ☐ × ☐ × ☐ = ☐

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## Basic Concepts Palindromes

A palindrome is a word or phrase that reads the same forward or backward.

Tie it!

Level

Racecar

Go, dog!

Was it a cat I saw?

A palindromic number is a symmetrical number, such as 1234321, which reads the same forward or backward.

**Example** Numbers that remain the same when their digits are reversed, such as 1881, are called palindromic numbers. Out of all 4-digit numbers, how many are palindromic numbers?

**Class Notes**

① Since the last two digits of a 4-digit palindromic number are the first two digits reversed, the number can be written as Ⓐ Ⓑ Ⓑ ☐.

② Since the first digit cannot be ☐, ☐ digits can be placed in Ⓐ and ☐ digits can be placed in Ⓑ.

③ Therefore, there are ☐ × ☐ = ☐ 4-digit palindromic numbers.

**Try It Again** How many 5-digit palindromic numbers are there?

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## Type 1-1 Palindromic Numbers

Numbers that remain the same when their digits are reversed, such as 55, 343 and 7227, are called palindromic numbers. How many palindromic numbers are there from 11 to 999?

**1** Out of all 2-digit numbers, how many are palindromic numbers?

**2** The first and last digits of 3-digit palindromic numbers are the same. How many digits can be placed in the hundreds place?

**3** From **2**, how many digits can be placed in the tens place?

**4** How many 3-digit palindromic numbers are there?

**5** How many palindromic numbers are there from 11 to 999?

SAMPLE



## Check Yourself

Key Points

Palindromic dates in January are 2-digit or 3-digit numbers.

**1** Numbers that remain the same when their digits are reversed, such as 44, 121 and 8558, are called palindromic numbers. If dates are expressed in the following way, how many dates in January and December can be expressed as palindromic numbers?

5 June → 65, 23 June → 623

**2** Digital clocks show time using four digits. Find out how many times the clock shows a palindromic time from 9 am to 6 pm.

Since digital clocks show time using four digits, 9 am is 0900 and 1 pm is 0100.

10 : 01 → 1001



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## Type 1-2 Numbers with Increasing Digits

From 100 to 200, find all numbers in which the ones digit is greater than the tens digit and the tens digit is greater than the hundreds digit.

**1** Excluding 200, which does not fit the conditions, the hundreds digit can only be 1. How many digits can be placed in the tens place?

**2** If 1 is in the hundreds place and 2 is in the tens place, how many digits can be placed in the ones place?

| Hundreds Place | Tens Place | Ones Place |
|---|---|---|
| 1 | 2 | |

**3** If 1 is in the hundreds place and 3 is in the tens place, how many digits can be placed in the ones place?

| Hundreds Place | Tens Place | Ones Place |
|---|---|---|
| 1 | 3 | |

**4** Using the same method, find all numbers that meet the conditions.

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## Check Yourself

Key Points

Think about the possibilities when the hundreds digit is a digit from 2 to 9.

**1** How many numbers are there in which the hundreds digit is greater than the tens digit and the tens digit is greater than the ones digit, as shown below?

763 321 975 641

**2** All dates from January to December are expressed as 4-digit numbers, as shown below. How many dates are there in which the digits are in increasing order from left to right?

The last two digits represent the day of the month. The greatest number that can be used is 29.

| 7 January | 29 October |
|---|---|
| 0107 | 1029 |



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**1** You can make different 5-digit numbers using the five number cards below. Find the difference between the second greatest number and the third smallest number.

| 7 | 0 | 3 | 9 | 6 |
|---|---|---|---|---|

**2** Numbers that remain the same when their digits are reversed, such as 404 and 2112, are called palindromic numbers. But a non-palindromic number can easily be changed into a palindromic number. Just reverse the number and add it to the original number, as shown below.
Numbers that become palindromic after step 1, such as 241, are called Step 1 palindromic numbers and numbers that become palindromic after 3 steps, such as 461, are called Step 3 palindromic numbers.

241 → 241 + 142 = 383
Step 1

461 → 461 + 164 = 625 → 625 + 526 = 1151 → 1151 + 1511 = 2662
Step 1 Step 2 Step 3

For each number below, find out how many steps are required to change them into a palindromic number.

(1) 378 → Step ☐ palindromic number (2) 831 → Step ☐ palindromic number
(3) 264 → Step ☐ palindromic number

SAMPLE



**3** John wants to make 3-digit numbers using three of the five number cards shown below. If he can use each card only once, how many different 3-digit odd numbers can he make?

| 5 | 2 | 6 | 9 | 8 |
|---|---|---|---|---|

**4** Use four number cards only once to make a 4-digit number. What is the hundredth greatest number?

| 0 | 1 | 4 | 6 | 8 | 9 |
|---|---|---|---|---|---|

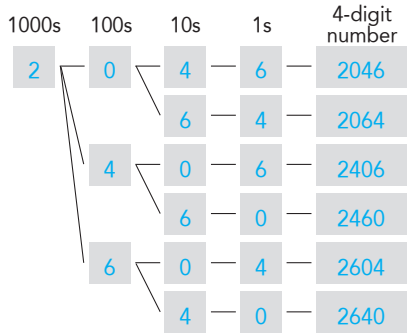
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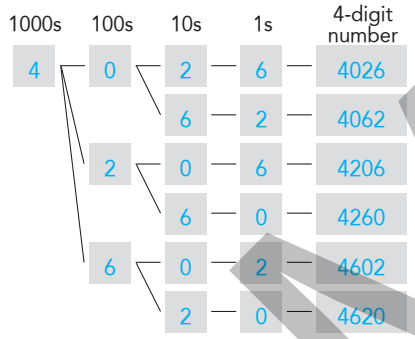
# Answer Key

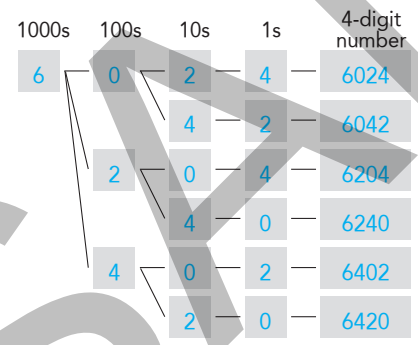
## I Number and Operations

### 1 Making Numbers with Number Cards p.8~p.9

Example ① 2, 4, 6




② 1000s 100s 10s 1s

3 × 3 × 2 × 1 = 18

Example ① A ② 0, 9, 10 ③ 9, 10, 90

Try It Again In the 5-digit palindromic number, ⓐ can be any of the nine digits except 0. ⓑ and ⓒ can be any of the ten digits. Therefore, there are 900 5-digit palindromic numbers.

### Type 1-1 Palindromic Numbers p.12~p.13

1 11, 22, 33, 44, 55, 66, 77, 88, 99
There are 9 palindromic numbers.

2 In the 3-digit palindromic number □△□, except 0, there are 9 digits (1 to 9) that can be placed in the □ .

3 In the 3-digit palindromic number □△□, there are 10 digits (0 to 10) that can be placed in the △.

4 There are 9 digits that can be placed in the hundreds place and the ones place, and 10 digits that can be placed in the tens place. Therefore, there are 9 × 10 = 90 3-digit palindromic numbers.

5 9 + 90 = 99 palindromic numbers

Check Yourself

1 January : 4 days (11, 111, 121, 131)
December: 2 days (121, 1221)

2 9 o'clock starts with 0900 and there is no such time as 0990; therefore, it does not have a corresponding time.
The times that are palindromic between 9 am and 6 pm, on a digital clock, are as shown below.

| 10:01 | 11:11 | 12:21 | 01:10 |
|---|---|---|---|
| 1001 | 1111 | 1221 | 0110 |

| 02:20 | 03:30 | 04:40 | 05:50 |
|---|---|---|---|
| 0220 | 0330 | 0440 | 0550 |

6 pm is displayed as 0600, so it is not a palindromic number. Therefore, there are 8 palindromic numbers.

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