

MATHS B

Modelling and Problem Solving

PHOENIX

SENIOR MATHS

PRACTICE QUESTIONS

BY TOPIC

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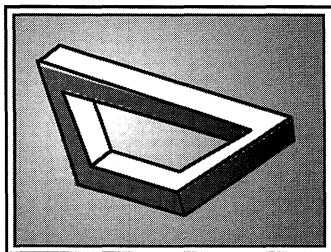
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CONTENTS

To the student	<i>iv</i>
Using a topic approach	<i>v</i>
Steps to success	<i>v</i>
Techniques for answering questions	<i>vi</i>
Exam techniques	<i>ix</i>
Abbreviations, symbols	<i>x</i>
Sample exam paper, with annotations	<i>xi</i>
Solutions to sample exam paper	131

TOPICS

1	Functions	1
2	Rates of change	12
3	Periodic functions & applications, and trigonometry	22
4	Exponential & logarithmic functions, and applications	51
5	Optimisation using derivatives	75
6	Integration	94
7	Applied statistical analysis	117

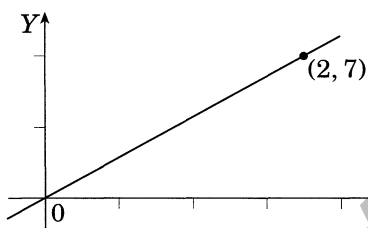


Topic 1

Functions

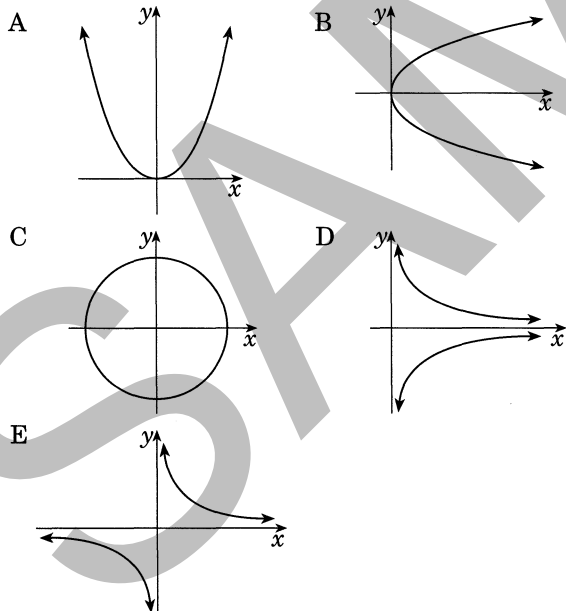
Examination questions

- Shade in the region on a number plane given by the two conditions $x^2 + y^2 \leq 4$ and $x + y \geq 1$.
- Find the range of the function
 $f: [0, \pi] \rightarrow \mathbb{R}, f(x) = 4\cos\left(x - \frac{\pi}{2}\right)$.
- A new set of axes on a cartesian plane is drawn. Instead of using the y axis and x axis, axis Y versus X^2 was chosen. This graph of Y against X^2 is shown below.

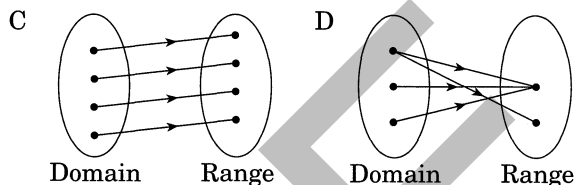
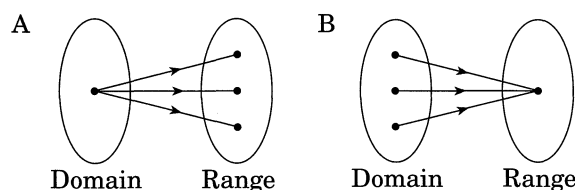


Find the equation of the function relating X and Y .

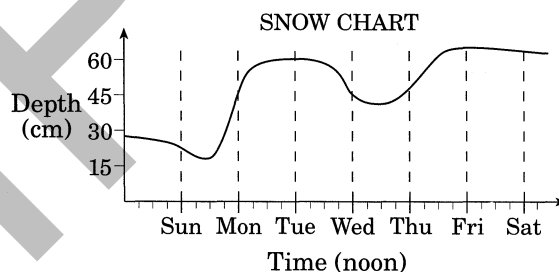
- Which of the graphs drawn below are functions? Give a reason.



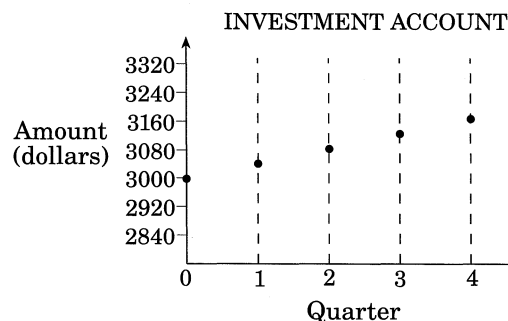
- Determine which of the following mappings are relations or functions.



- Which of the following are functions? Justify your answer.
 A $(3, 6) (4, 6) (5, 9) (7, 11) (7, 13)$
 B $(1, 5) (2, 5) (3, 5) (4, 5) (5, 5)$
 C $(x, 2\sqrt{x})$ where $x \in \mathbb{R}^+$ and $y \in \mathbb{R}$
- The manager of a ski-lodge keeps a record of the depth of snow outside the lodge throughout the ski season. The graph below shows data collected during one week last September.



- On which days during the week did it snow?
 - One day during the week it rained all day. Which day was this?
- In 1995, a certain number of rabbits were on Flinders Island. From that time the rabbit population increased by 18% every year. After three years the number of rabbits on the island was 7856. How many rabbits were on Flinders Island in 1995?
 - The graph below shows the amount of money in an investment account when simple interest is paid on the original amount invested. This original amount is \$3000 and interest is credited to the account quarterly. Calculate the annual simple interest rate.



- 10 How does the transformation $y = f(-x)$ affect each of the following functions?

$$\begin{aligned}y_1 &= x^3 \\y_2 &= x^2 \\y_3 &= \sin x \\y_4 &= \cos x\end{aligned}$$

Use sketches to explain your answer.

- 11 For a research project thousands of babies were weighed at birth and then on each of the 14 days after their birth. The researchers found that the average weight at birth was 3.27 kg. A mathematical formula was developed for the weight of an average baby on each 14th day after birth.

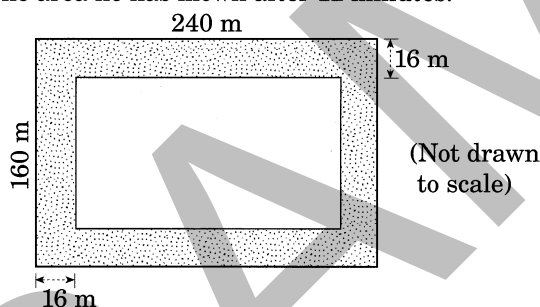
If t represents the number of days after birth and w is the weight of the baby in kilograms, the formula is $w = 0.015t^2 - 0.18t + 3.27$.

According to this formula, how many days will it take for an average baby to return to its birth weight?

- 12 Two cars each travel a distance of 175 km. Car Corona has a fuel consumption rate that allows it to travel 4 km more per litre of fuel compared to car Commodore. Car Commodore uses 5 litres more than car Corona during their respective journeys.

By setting up an appropriate model (equation), calculate the consumption rate of car Commodore.

- 13 Merv has to mow a rectangular area 240 metres long and 160 metres wide. He mows a series of rectangular strips. The shaded area below shows the area he has mown after 12 minutes.



How long will it take Merv to mow the entire area?

- 14 A person dies and leaves a total of \$470 700 to be divided equally among all the people named in the will. Unfortunately 3 people named in the will die before it is settled. This increases each of the remaining people's share by \$26 150.

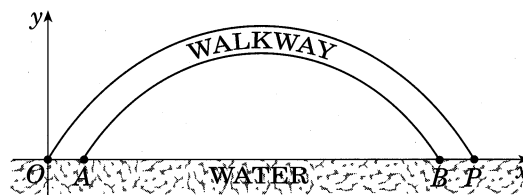
Find the number of people originally named in the will.

- 15 The top and bottom of a solid concrete arch walkway to cross a waterway are modelled by a pair of quadratic functions

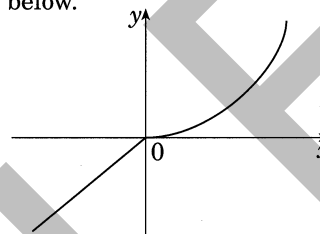
$$\begin{aligned}f(x) &= 3x - 0.1x^2 \\g(x) &= -0.1x^2 + 3x - 8.1\end{aligned}$$

where x is measured in metres.

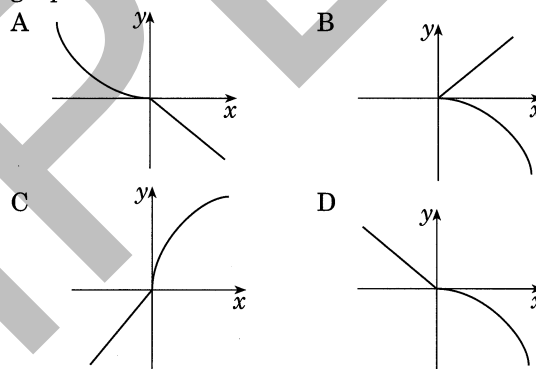
The x axis, $OABP$, is ground level and y measures the height (in metres) above ground level.



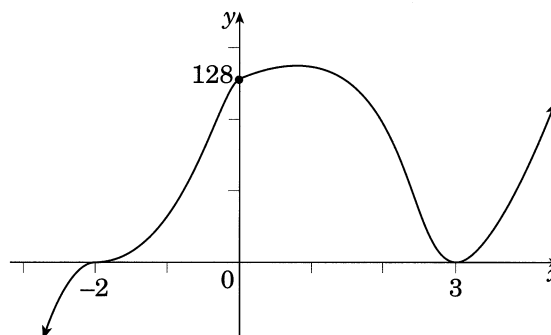
- (a) Find the width of the waterway spanned by the arch.
(b) Find the thickness of the concrete in the walkway, at its highest point.
- 16 The graph of the function with equation $y = f(x)$ is shown below.



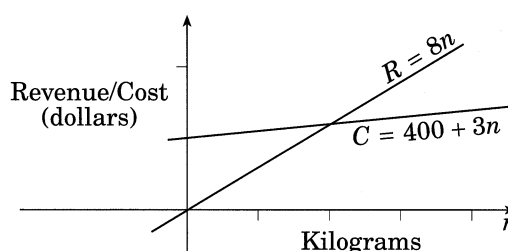
Which of the following is most likely to be the graph of the inverse function?



- 17 Find a possible equation for the graph shown below.

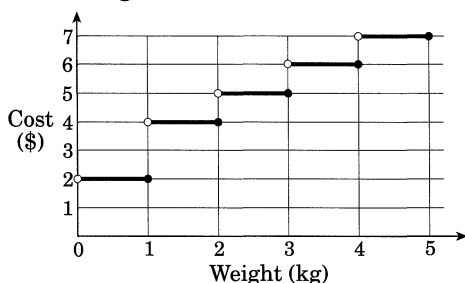


- 18 R is the revenue, in dollars, earned by a strawberry farmer selling n kilograms of strawberries. C is the cost, in dollars, of producing n kilograms of strawberries.



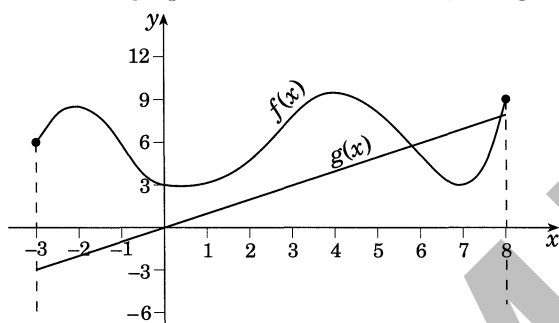
Calculate the number of kilograms of strawberries that the farmer needs to produce before he starts to make a profit.

- 19 The graph below shows the cost of posting parcels of various weights.



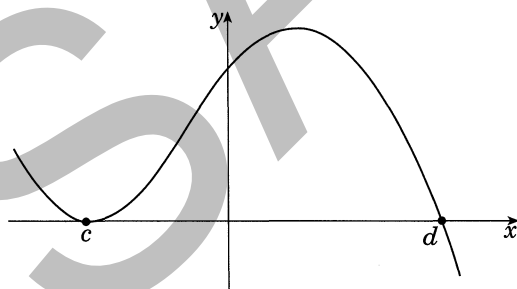
If 2 parcels are posted, weighing 2.5 kg and 4 kg respectively, find the total cost to send both parcels.

- 20 Two functions $f(x)$ and $g(x)$ are drawn below. Sketch the graph of $r(x)$ where $r(x) = f(x) + g(x)$.



- 21 The water supply to a large farm is kept in two storage tanks labelled A and B. Tank A holds 800 litres when full. Water flows into this tank at a rate of 25 litres per minute. Tank A is initially empty. Tank B holds 1000 litres of water when full. Water flows into tank B at a rate of 200 mL per second. Initially there are 250 litres of water in tank B.
- (a) How long after tank A is full will tank B be full?
- (b) When will the volume of each tank be the same?

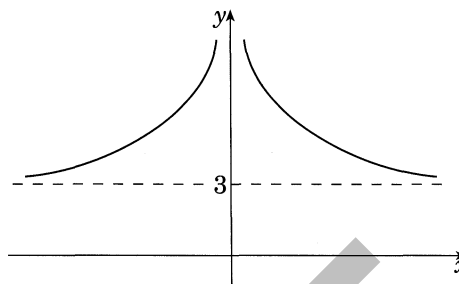
22



The graph shown could be that of a function f with a rule of:

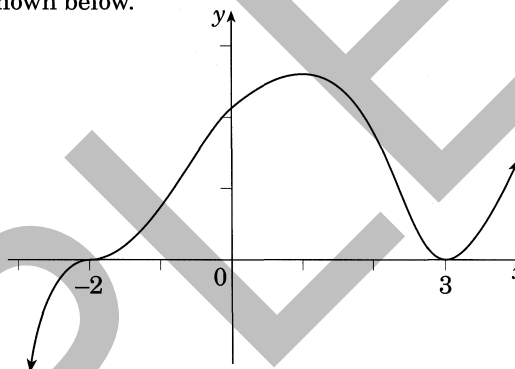
- A $f(x) = (x+c)^2(d-x)$
 B $f(x) = (x-d)^2(c-x)$
 C $f(x) = -(x-c)^2(x+d)$
 D $f(x) = (x-c)^2(x-d)$
 E $f(x) = (x+c)^2(x-d)$

- 23 The graph with equation $y = f(x)$ is shown below.

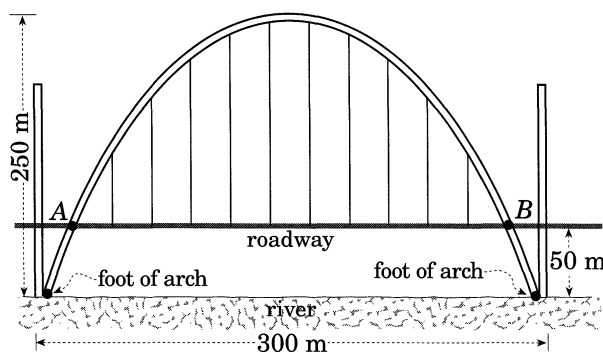


Sketch the graph with equation $y = 1 - f(x)$.

- 24 Find a possible equation that satisfies the graph shown below.



- 25 The bacteria count per cm^3 of water, B , is given by the formula $B = 30t^2 - 240t + 500$, where t = number of days.
- (a) Find the initial bacteria count.
- (b) After how many days will the bacteria count be 140 per cm^3 ?
- 26 A bridge is constructed with the design in the shape of a parabola as shown below.



The distance from the top of the bridge to the river is 250 metres. The river is 50 metres below the roadway running through the bridge. The 'feet' of the bridge's arch is anchored at the level of the river.

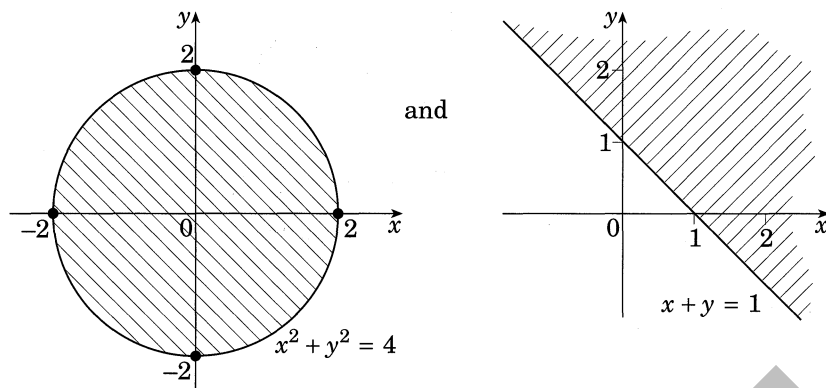
Ten vertical supports are to be placed equally between the road and arch (A to B as shown). The arch spans a distance of 300 m.

Find the distance between adjacent supports.

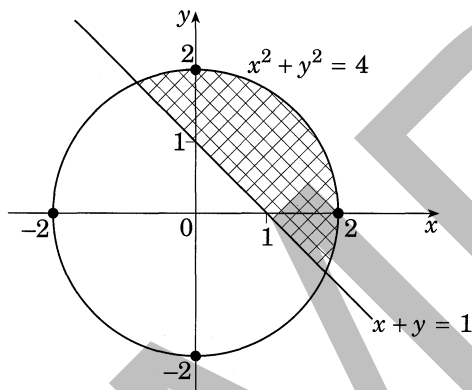
WORKED SOLUTIONS

- 1 $x^2 + y^2 \leq 4 \Rightarrow$ region on and inside the circle centre (0,0), radius = 2
 $x + y \geq 1 \Rightarrow y \geq -x + 1$

This region is above and on the line $y = -x + 1$.



Both conditions are satisfied by the common region shown in the diagram below.



- 2 Let $x = 0 \therefore f(0) = 4 \cos\left(0 - \frac{\pi}{2}\right) = 4 \cos \frac{3\pi}{2} = 4 \times 0 = 0$
 Let $x = \pi \therefore f(\pi) = 4 \cos\left(\pi - \frac{\pi}{2}\right) = 4 \cos \frac{\pi}{2} = 4 \times 0 = 0$
 Let $x = \frac{\pi}{2}$ [a point within the domain $(0, \pi)$]
 $\therefore f\left(\frac{\pi}{2}\right) = 4 \cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 4 \cos 0 = 4 \times 1 = 4$
 $\therefore$ Range = $(0, 4)$

- 3 Gradient of the line = $\frac{7-0}{2-0} = \frac{7}{2}$
 $\therefore$ Equation is $y = \frac{7}{2} \cdot x^2$

- 4 Functions: A, E

Reason: For graphs A and E, for each x value there is only one unique y value. That is for every x value on the graph, we can only find one y point on the graph. [This can be seen by applying the vertical line test.]

- 5 Functions: B, C

Relations: A, D

Justification: To be a function, every x value must have only **one** y value corresponding to it.

In (A), the x point has 3 different y values.

In (D), the first x point has 2 different y values.

$\therefore$ A, D are relations and B, C are functions

Point $x = \frac{\pi}{2}$ is a turning point.

Since $f(x) = 4 \cos\left(x - \frac{\pi}{2}\right)$,

then $f'(x) = -4 \sin\left(x - \frac{\pi}{2}\right)$.

Turning point $\Rightarrow f'(x) = 0$

$\therefore -4 \sin\left(x - \frac{\pi}{2}\right) = 0$

$\therefore \sin\left(x - \frac{\pi}{2}\right) = 0$

Hence $x = \frac{\pi}{2}$ since $\sin 0 = 0$.

- 6 A is **not** a function since the x value of 7 has 2 different y values, i.e. (7, 11) (7, 13).
 B is a function as each x value has only 1 y value.
 C is **not** a function as the square root of x can be both positive and negative.
 e.g. if $x = 1$, $y = 2 \times \sqrt{1} = 2 \times \pm 1 = 2$ or -2
 if $x = 4$, $y = 2 \times \sqrt{4} = 2 \times \pm 2 = 4$ or -4

The y value can be positive or negative since $y \in \mathbb{R}$

- 7 (a) The depth of the snow increased from midnight Sunday and midnight Wednesday. Therefore it snowed on Monday and Thursday.
 (b) The depth of the snow decreased from midnight Tuesday, therefore it rained on Wednesday.

- 8 Let the number of rabbits on the island in 1995 = x
 1 year later, in 1996, the number = $x + 18\%$ of x
 $= x + 0.18x$
 $= 1.18x$
 2 years later, in 1997, the number = $1.18x + (18\% \text{ of } 1.18x)$
 $= 1.18x + (0.18 \times 1.18x)$
 $= 1.18x + 0.2124x$
 $= 1.3924x$
 3 years later, in 1998, the number = $1.3924x + (18\% \text{ of } 1.3924x)$
 $= 1.3924x + 0.250632x$
 $= 1.643032x$
 $\therefore 1.643032x = 7856$
 $x = 7856 \div 1.643032$
 $x = 4781.404136$
 $\therefore$ No. of rabbits in 1995 = 4781.

Alternative method

Using the formula for a geometric progression (GP):

$$T_n = a \cdot r^{n-1}$$

$$a = x, r = 1.18, n = 4$$

$$x \times 1.18^3 = 7856$$

$$\therefore x = \frac{7856}{1.18^3}$$

$$\div 4781$$

- 9 Principal (P) = \$3000
 Final amount (A) = $P + P \times i \times n$, where i = interest rate/100
 n = no. of years

$$\therefore A = P(1 + in)$$

$$3160 = 3000(1 + i \times 1)$$

$$\frac{3160}{3000} = (1 + i)$$

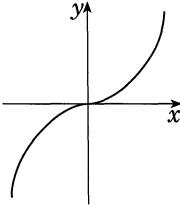
$$1.053 = 1 + i$$

$$0.053 = i$$

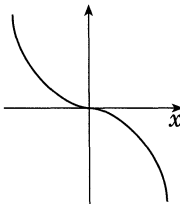
$$\therefore 5.3\% = \text{Interest rate}$$

$$\left(5\frac{1}{3}\%\right) = \text{Interest rate}$$

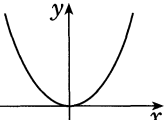
10 $y_1 = x^3$
 $f(-x) = (-x)^3 = -x^3 \Rightarrow f(x) = x^3 \Rightarrow$



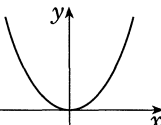
$f(-x) = -x^3 \Rightarrow$



$y_2 = x^2$
 $f(-x) = (-x)^2 = x^2 \Rightarrow f(x) = x^2$



$f(-x) = x^2 \Rightarrow$



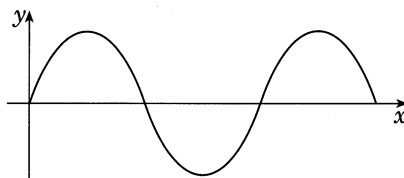
Points reflect on the x axis.

$$y_3 = \sin x \quad f(x) = \sin(-x)$$

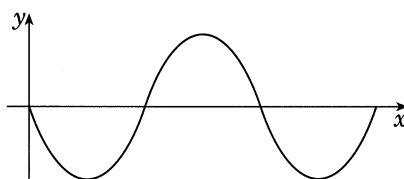
$$= \sin(2\pi - x)$$

$$= -\sin x$$

$$\Rightarrow f(x) = \sin x$$



$$\Rightarrow f(-x) = -\sin x$$

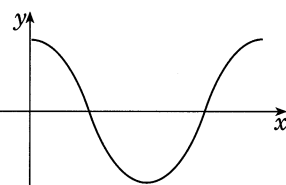


$$y_4 = \cos x \quad f(-x) = \cos(-x)$$

$$= \cos x$$

$$\Rightarrow f(x) = \cos x$$

$$f(-x) = \cos x$$



11 Original birth weight = 3.27

$$\therefore \text{if } w = 0.015t^2 - 0.18t + 3.27$$

$$\Rightarrow 3.27 = 0.015t^2 - 0.18t$$

$$\therefore 0 = 0.015t^2 - 0.18t$$

$$0 = t(0.015t - 0.18)$$

$$\therefore \text{Either } t = 0 \text{ or } 0.015t = 0.18$$

$$t = \frac{0.18}{0.015}$$

$$t = 12$$

Since $t = 0 \Rightarrow$ day of birth, then $t = 12$ is the only solution,

$\therefore$ 12 days after birth.

12 Let the consumption rate of car Commodore = x km/L

$\therefore$ the consumption rate of car Corona = $(x + 4)$ km/L

No. of litres used in the journey by Commodore = $\frac{175}{x}$

No. of litres used in the journey by Corona = $\frac{175}{x+4}$

Now since Commodore is using 5 more litres,

$$\frac{175}{x} - \frac{175}{x+4} = 5$$

$$\therefore \frac{175(x+4) - 175x}{x(x+4)} = 5$$

$$175(x+4) - 175x = 5x(x+4) \quad [\text{cross-multiply}]$$

$$175x + 700 - 175x = 5x^2 + 20x$$

$$700 = 5x^2 + 20x$$

$$\therefore 0 = 5x^2 + 20x - 700$$

$$\therefore 0 = x^2 + 4x - 140$$

Now solving our quadratic: $0 = (x+14)(x-10)$

$$\therefore x = -14 \text{ or } x = 10$$

Since $x = -14$ is invalid (since consumption is not negative)

$$\Rightarrow x = 10$$

$\therefore$ Commodore's consumption rate = 10 km/L.

13 Shaded area already mown = $(240 \times 16 \times 2) + [(160 - 2 \times 16) \times 16 \times 2]$

$$= 7680 + 4096$$

$$= 11\,776 \text{ m}^2$$

$\therefore$ Area 11 776 m² took 12 minutes

$$\Rightarrow 1 \text{ m}^2 \text{ took } \frac{12}{11\,776} \text{ minutes}$$

Alternative solution for quadratic:

$$0 = x^2 + 4x - 140$$

$$a = 1, \quad b = 4, \quad c = -140$$

$$x = \frac{-b \pm \sqrt{b^2 - (4ac)}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - (4 \times 1 \times -140)}}{2}$$

$$x = \frac{-4 \pm \sqrt{576}}{2}$$

$$x = \frac{-4 \pm 24}{2}$$

$$x = \frac{-4 + 24}{2} \quad \text{or} \quad x = \frac{-4 - 24}{2}$$

$$x = \frac{20}{2} \quad x = -\frac{28}{2}$$

$$x = 10 \quad x = -14$$

continued ...

$$\begin{aligned}\text{Total area} &= 240 \text{ m} \times 160 \text{ m} \\ &= 38\,400 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{Time for total area} &= \left(38\,400 \times \frac{12}{11\,776} \right) \text{ minutes} \\ &= 39.13 \text{ minutes}\end{aligned}$$

14 Let x = original no. of people named in the will.

$$\therefore \text{Share of each person} = \frac{470\,700}{x}$$

No. of people remaining to share will = $x - 3$

$$\therefore \text{Share of remaining person} = \frac{470\,700}{x-3}$$

$$\therefore \frac{470\,700}{x-3} - \frac{470\,700}{x} = 26\,150$$

$$\frac{470\,700x - 470\,700(x-3)}{x(x-3)} = 26\,150$$

$$\therefore \frac{470\,700x - 470\,700x + 1\,412\,100}{x(x-3)} = 26\,150$$

$$\therefore 1\,412\,100 = 26\,150x(x-3)$$

$$1\,412\,100 = 26\,150x^2 - 78\,450x$$

$$0 = 26\,150x^2 - 78\,450x - 1\,412\,100$$

$$0 = 2615x^2 - 7845x - 141\,210$$

$$0 = 523x^2 - 1569x - 28\,242 \quad \leftarrow$$

$$a = 523, \quad b = -1569, \quad c = -28\,242$$

$$x = \frac{-b \pm \sqrt{b^2 - (4ac)}}{2a}$$

$$x = \frac{1569 \pm \sqrt{1569^2 - (4 \times 523 \times -28\,242)}}{2 \times 523}$$

$$x = \frac{1569 \pm \sqrt{61\,544\,025}}{1046}$$

$$x = \frac{1569 \pm 7845}{1046}$$

$$x = \frac{(1569 - 7845)}{1046} = -6 \quad (\text{not possible})$$

$$\text{or} \quad x = \frac{(1569 + 7845)}{1046} = 9$$

$\therefore$ Original number of people = 9

Cross-multiplying

This is a quadratic equation.

15 $f(x) = 3x - 0.1x^2$ OR

$$\therefore f(x) = -0.1x^2 + 3x$$

$$f(x) = -0.1x(x-30)$$

$$\therefore -0.1 = 0 \quad \text{or} \quad x - 30 = 0$$

$$\therefore x = 0 \quad \quad \quad x = 30$$

$$f(x) = -0.1x^2 + 3x$$

$$a = -0.1, \quad b = 3, \quad c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - (4ac)}}{2a}$$

$$x = \frac{-3 \pm \sqrt{9 - (4 \times -0.1 \times 0)}}{-0.2}$$

$$x = \frac{-3 \pm 3}{-0.2}$$

$$x = \frac{-3 + 3}{-0.2} \quad \text{or} \quad x = \frac{-3 - 3}{-0.2}$$

$$x = 0 \quad \quad \quad x = \frac{-6}{-0.2} = 30$$

We let $f(x)$ then $g(x)$ equal zero to find where the graph cuts the x axis.

$$\begin{aligned}\therefore g(x) &= -0.1x^2 + 3x - 8.1 \\ &= -0.1(x^2 - 30x + 81)\end{aligned}$$

$$= -0.1(x-27)(x-3)$$

$$\therefore x = 27 \quad \text{or} \quad x = 3$$

OR $a = -0.1$, $b = 3$, $c = -8.1$

$$x = \frac{-3 \pm \sqrt{9 - (4 \times -0.1 \times -8.1)}}{-0.2}$$

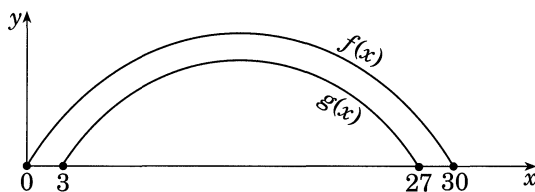
$$x = \frac{-3 \pm \sqrt{5.76}}{-0.2}$$

$$x = \frac{-3 \pm 2.4}{-0.2}$$

$$x = \frac{(-3 - 2.4)}{-0.2} \quad \text{or} \quad x = \frac{(-3 + 2.4)}{-0.2}$$

$$x = \frac{-5.4}{-0.2} \quad x = \frac{-0.6}{-0.2}$$

$$x = 27 \quad x = 3$$



(a) Waterway = $27 - 3 = 24$ metres

(b) Highest point on $f(x)$ is the turning point.

$$x = \frac{-b}{2a} = \frac{-3}{-0.2} = 15 \text{ m}$$

$$\begin{aligned} \therefore y &= 3 \times 15 - 0.1 \times 15 \times 15 \\ &= 45 - 22.5 \\ &= 22.5 \text{ m} \end{aligned}$$

Highest point on $g(x)$ is the turning point.

$$x = \frac{-b}{2a} = \frac{-3}{-0.2} = 15 \text{ m}$$

$$\begin{aligned} \therefore y &= -0.1 \times 15^2 + 3 \times 15 - 8.1 \\ &= -22.5 + 45 - 8.1 \\ &= 14.4 \text{ m} \end{aligned}$$

$$\therefore \text{Thickness of highest point} = 22.5 - 14.4 = 8.1 \text{ m}$$

16 Graph C, since reflection in line $y = x$ produces shape C.

17 Graph shows product of 2 possible shapes being $x^{\text{odd power}}$ and $x^{\text{even power}}$.

At $x = -2$, graph has shape of $x^{\text{odd power}} \Rightarrow (x+2)^3$

At $x = 3$, graph has shape of $x^{\text{even power}} \Rightarrow (x-3)^2$

$$\therefore y = A(x+2)^3(x-3)^2$$

Let $x = 0$, $y = A \times 2^3 \times (-3)^2$

$$= 72A$$

$$\therefore 128 = 72A$$

$$\frac{16}{9} = \frac{128}{72} = A$$

$$\therefore \text{Possible equation is } y = \frac{16}{9}(x+2)^3(x-3)^2$$

18 To make a profit, revenue - cost > 0

$$\therefore 8n - (400 + 3n) > 0$$

$$8n - 400 - 3n > 0$$

$$5n > 400$$

$$n > 80$$

$\therefore$ If the farmer produces more than 80 kg of strawberries, he will make a profit.

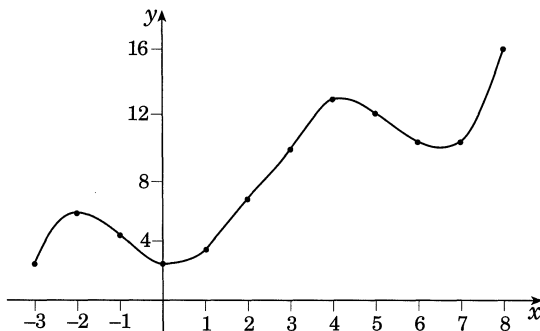
19 2.5 kg parcel costs \$5

4 kg parcel costs \$6 (as 4 kg weight is a closed circle on the graph)

$\therefore$ Total = \$11

20 Choosing some points:

x	-3	-2	-1	0	1	2	3	4	5	6	7	8
From graph $f(x)$	6	8	6	3	3	5	8	10	8	5	4	9
From graph $g(x)$	-3	-2	-1	0	1	2	3	4	5	6	7	8
$r(x) = f(x) + g(x)$	3	6	5	3	4	7	11	14	13	11	11	17



21 (a) Let a = no. of minutes to fill tank A

$$\therefore 25a = 800$$

$$a = 800 \div 25$$

$$a = 32 \text{ minutes}$$

Let b = no. of seconds to fill tank B

$$\therefore 0.2b = 1000 - 250$$

$$= 750$$

$$b = 750 \div 0.2$$

$$= 3750 \text{ seconds}$$

$$\therefore b = 62.5 \text{ minutes}$$

$$\therefore \text{Tank B takes a further } 62.5 - 32 = 30.5 \text{ minutes.}$$

(b) $V_A = 25t$ where V_A = volume of tank A

t = no. of minutes of filling

$$V_B = 250 + 0.2 \times 60 \times t \text{ where } V_B = \text{volume of tank B}$$

$$= 250 + 12t$$

t = no. of minutes of filling

$$\therefore V_A = V_B \text{ when volume is the same}$$

$$\therefore 25t = 250 + 12t$$

$$13t = 250$$

$$t = \frac{250}{13}$$

$$t = 19.23 \text{ minutes}$$

$$\therefore t \div 19 \text{ minutes } 13.8 \text{ seconds}$$

22 If we look at point c , it has been translated to the left from the general shape of the curve x^2 . Therefore this part of the graph is $(x+c)^2$. Point d has translated right from shape $y = -x$.

Therefore this part of the graph is $-(x-d) = (d-x)$.

$$\text{Hence A: } f(x) = (x+c)^2(d-x)$$

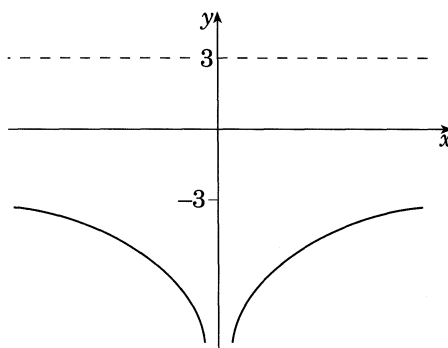
23 $y = 1 - f(x)$

$$\therefore y = -f(x) + 1$$

$\therefore$ We need to make the original graph negative for all y values, then add 1.

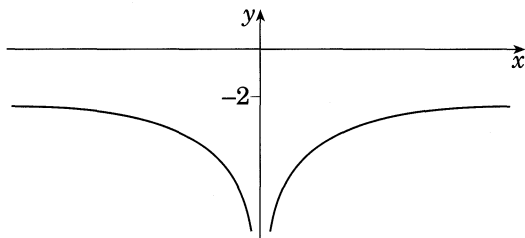
Step 1:

$$y = -f(x) \Rightarrow$$



Step 2: $y = -f(x) + 1$

Therefore we need to add 1 to every y value.



- 24** Possible equation $y = A(x+2)^3 \times (x-3)^2$
 since shape at $x = -2$ is in the form $y = x^{\text{odd power}}$
 and shape at $x = 3$ is in the form $y = x^{\text{even power}}$.
 $\therefore$ If $y = A(x+2)^3 \times (x-3)^2$ cuts the y axis when $x = 0$,
 $\therefore y = A \times (0+2)^3 \times (0-3)^2$
 $= A \times 8 \times 9$
 $144 = 72A$
 $2 = A$
 $\therefore$ Possible equation: $y = 2(x+2)^3 \times (x-3)^2$

- 25 (a)** Initial count $\Rightarrow t = 0$, where $B = 30t^2 - 240t + 500$

$$\therefore B = 30 \times 0^2 - 240 \times 0 + 500$$

$$\therefore B = 500$$

- (b)** $140 = 30t^2 - 240t + 500$
 $0 = 30t^2 - 240t + 500 - 140$
 $= 30t^2 - 240t + 360$
 $= t^2 - 8t + 12$

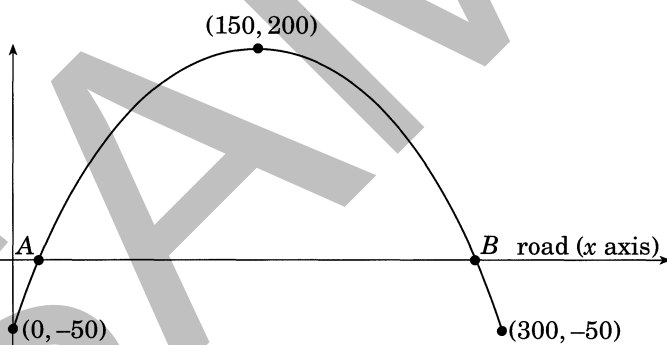
$$\therefore 0 = (t-2)(t-6)$$

$$\therefore t = 2 \text{ days or } t = 6 \text{ days}$$

$t^2 - 8t + 12$ factorises by choosing two numbers that ADD to -8 and MULTIPLY to 12 , ie. -2 and -6 .

- 26 Step 1:** Find the equation of the parabola.

Step 2: Find where the parabola cuts the x axis, i.e. our roadway.



Let the parabola be $y = ax^2 + bx + c$.

Using points on the curve:

$$(0, -50) \quad -50 = a \times 0^2 + b \times 0 + c$$

$$-50 = c$$

$$\therefore y = ax^2 + bx - 50$$

$$(300, -50) \quad -50 = a \times 300^2 + b \times 300 - 50$$

$$0 = 300 \times 300a + 300b$$

$$\therefore 0 = 300a + b$$

$$\therefore b = -300a$$

$$\therefore y = ax^2 - 300ax - 50$$

$$(150, 200) \quad 200 = a \times 150^2 - 300a \times 150 - 50$$

$$250 = a \times 150 \times 150 - 300 \times 150a$$

$$= 22\,500a - 45\,000a$$

continued . . .

$$250 = -22500a$$

$$\frac{250}{-22500} = a$$

$$-\frac{1}{90} = a$$

$$\begin{aligned}\therefore b &= -300 \times a \\ &= -300 \times -\frac{1}{90} \\ &= \frac{300}{90} \\ &= \frac{10}{3}\end{aligned}$$

Equation of parabola: $y = -\frac{1}{90}x^2 + \frac{10}{3}x - 50$

Cuts x axis if $y = 0$, $\therefore 0 = -\frac{1}{90}x^2 + \frac{10}{3}x - 50$

$$a = -\frac{1}{90}, \quad b = \frac{10}{3}, \quad c = -50$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-\frac{10}{3} \pm \sqrt{\frac{100}{9} - \left(4 \times -\frac{1}{90} \times -50\right)}}{-\frac{1}{45}}$$

$$\begin{aligned}x &= \frac{-3.\dot{3} \pm \sqrt{8.\dot{8}}}{-\frac{1}{45}} \\ &= \frac{-3.\dot{3} \pm 2.98}{-0.0\dot{2}}\end{aligned}$$

$$\therefore x = \frac{-3.\dot{3} + 2.98}{-0.0\dot{2}} \quad \text{or} \quad x = \frac{-3.\dot{3} - 2.98}{-0.0\dot{2}}$$

$$x = 17.\dot{6} \quad \text{or} \quad x = 284.1$$

$$\therefore A = (17.\dot{6}, 0), \quad B = (284.1, 0)$$

$$\text{Distance } A \rightarrow B = 284.1 - 17.\dot{6} = 266.43$$

$$\therefore \text{Distance between adjacent supports} = \frac{266.43}{11} = 24.22 \text{ m}$$

We need to divide by 11, since there are 11 spaces between the 10 supports.