MATHEMATICAL OLYMPIAD TRAINING BOOK

LEVEL 6

(12–13 years)

Name: ____________________

Class: ____________________

Terry Chew

Master trainer at Terry Chew Academy
and Co-Founder/Academic Director of
SEAMO
SOUTHEAST ASIAN MATHEMATICAL OLYMPIAD

Mathematical Olympiad Training Book Level 6

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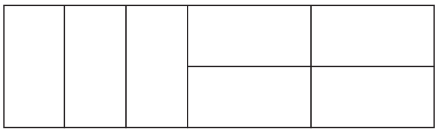
1. Seven identical dominoes of size 1 cm by 2 cm and with identical faces on both sides are arranged to cover a rectangle of size 2 cm × 7 cm. One possible arrangement is shown below. Find the total number of ways in which the rectangle can be covered by the seven dominoes.



2. Among 64 students, 28 of them like Science, 41 like Mathematics and 20 like English. 24 of them like both Mathematics and English. 12 students like both Science and English. 10 students like both Science and Mathematics. How many students like all the three subjects?

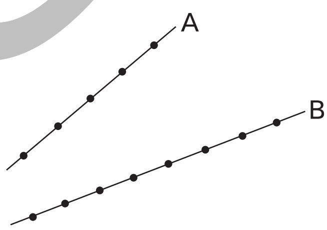
3. What is the value of the digit in the ones place of the following?

$1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times \ldots \times 2017 \times 2019$

4. Evaluate

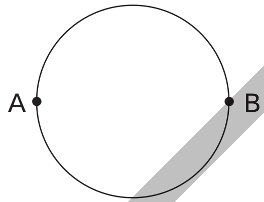
$$\frac{1+2+3+4+5+6+7+6+5+4+3+2+1}{7777777 \times 7777777}.$$

5. There are 5 dots on line A and 8 dots on line B.



Find the total number of triangles that can be formed using any 3 dots as their vertices.

6. Jonathan and Cindy run on a circular track where AB is the diameter of the track, as shown below.



If Jonathan and Cindy run towards each other at the same time from Point A and Point B respectively, it will take them 40 seconds before they meet. If they start running at the same time but in the same direction, it will take Jonathan 280 seconds to catch up with Cindy. What is the ratio of their speeds?

7. Find the value of

$1^2 - 2^2 + 3^2 - 4^2 + \cdots + \quad + 2017^2 - 2018^2.$

8. Between 1 and 2018, how many numbers are multiples of 5 or 7?

9. The sum of the digits of a 3-digit number is 18. The tens digit is 1 more than the ones digit. If the hundreds digit and the ones digits are swapped, the difference between the new number and the original number is 396.
What is the original number?

10. Evaluate $\left(1+\frac{1}{3}+\frac{1}{5}\right)\times\left(\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\right)-\left(1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\right)\times\left(\frac{1}{3}+\frac{1}{5}\right)$ using a simple method.

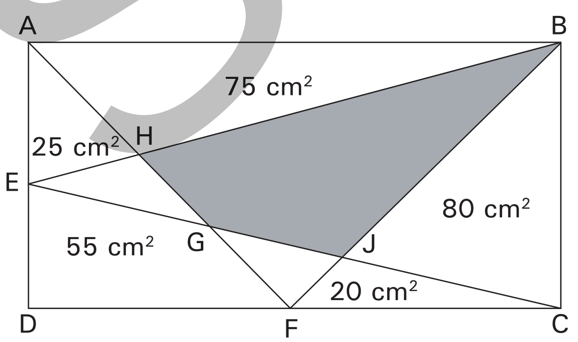
11. **(a)** Choose any three letters from a, b, c, and d. In how many ways can we arrange the three letters?

(b) A teacher wants to choose a captain and vice-captain among 12 volleyball players. In how many ways can he do so?

12. A car travelled to Town B from Town A at a constant speed of 72 km/h. It then returned from Town B to Town A at a constant speed of 48 km/h. What was the average speed of the car for the whole journey?

13. Lucas multiplies his month of birth by 31. He then multiplies his day of birth by 12. The sum of the two products is 213. When is his birthday?

14. In the figure below, E and F are midpoints of AD and DC respectively. ABCD is a rectangle. Find the area of the shaded region.



15. Find the value of the following.

$20172018 \times 20182017 - 20172017 \times 20182018$

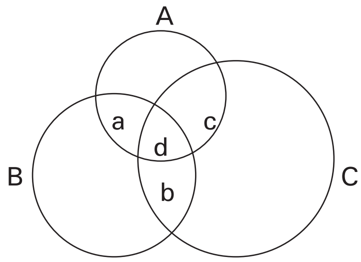
16. Evaluate $29\frac{1}{2}\times\frac{2}{3}+39\frac{1}{3}\times\frac{3}{4}+49\frac{1}{4}\times\frac{4}{5}$.

17. How many 3-digit numbers have the sum of the three digits equalling to 4?

18. A car will travel from Town A to Town B. If it travels at a constant speed of 60 km/h, then it will arrive at 3.00 pm. If it travels at a constant speed of 80 km/h, then it will arrive at 1.00 pm. At what speed should it be travelling if the driver aims to arrive at Town B at 2.00 pm?

19. A big box can hold 48 marbles. A small box can hold 30 marbles. Find the number of big boxes and the number of small boxes that can hold a total of 372 marbles.

20. In the figure below, the area of three circles, A, B and C, are 40 cm^2, 50 cm^2 and 60 cm^2 respectively. Given that a + d = 12 cm^2, b + d = 14 cm^2, c + d = 16 cm^2 and d = 8 cm^2, find the area of the whole figure.



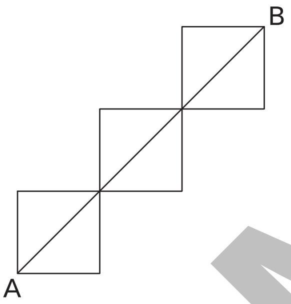
21. The sum of two numbers is 88. The product of the two numbers is 1612. What are the two numbers?

22. Evaluate

$$\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)$$

using a simple method.

23. How many ways are there to reach B from A? Only movements →, ↑ and ↗ are allowed.



24. A tourist was travelling to a town that was 60 km away. He walked at a speed of 6 km/h at first. Then, he hitched a ride on a scooter travelling at 18 km/h. He arrived at the town 4 hours from the time he set off. How long had he walked?

25. Two fruit baskets contain some oranges. If an orange is transferred from the first basket to the second basket, then both baskets will have the same number of oranges. If an orange is transferred from the second basket to the first basket, then the number of oranges in the first basket becomes thrice the number of oranges in the second basket. How many oranges are in each basket at first?

26. A survey was conducted on 250 students regarding their preferred school activities: badminton, volleyball and basketball. 140 of them liked badminton, 120 of them liked volleyball and 100 of them liked basketball. 40 of them liked both badminton and volleyball, but not basketball. 20 of them liked badminton and basketball, but not volleyball. How many liked both volleyball and basketball, but not badminton, given 10 liked all three activities?

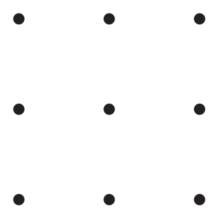
27. A contractor has 1088 square tiles. In how many ways can he form a rectangle using all the tiles each time?

28. Evaluate

$$\frac{1\times2\times3+2\times4\times6+3\times6\times9+\ldots+100\times200\times300}{1\times3\times5+2\times6\times10+3\times9\times15+\ldots+100\times300\times500}$$

by factorising.

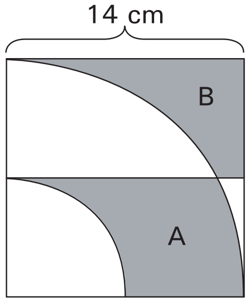
29. In the figure below, how many triangles can be formed using any three points as the vertices?



30. A fighter plane had enough fuel to last a 6-hour flight. The speed of wind and the speed of the plane made up a total of 1500 km/h when the plane was flying in the direction of the wind during its mission. On its return trip, the total speed was reduced to 1200 km/h as the plane was travelling against the wind. How far could the plane travel before it made its return?

31. Julie asked her teacher, "How old were you in 2008?" "My age in 2008 was the sum of all the digits of my year of birth," replied the teacher. How old was the teacher in 2008?

32. In the figure below, the side of the square is 14 cm. The radii of the two quadrants are 7 cm and 14 cm respectively. A and B represent the areas of the two shaded regions. Find (A – B). $\left(\text{Take } \pi = \frac{22}{7}.\right)$



33. For $1^2 + 2^2 + 3^2 + \ldots + n^2$, we can compute using $n(n + 1)(2n + 1) \div 6$. Find the value of $1^2 + 2^2 + 3^2 + \ldots + 15^2$.

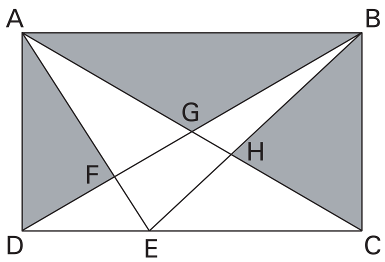
34. Evaluate $\frac{12345678}{12345678^2 - 12345677 \times 12345679}$.

35. Let $m = 2^n - n^2$, where $n = 1, 2, 3, \ldots, 2019$. How many values of m are there so that m is divisible by 6?

36. During a school walkathon, Alan completed the first half of the journey at a speed of 4.5 km/h. He then finished the second half of the journey at a speed of 5.5 km/h. On the other hand, Benny walked at a speed of 4.5 km/h for the first half of the time taken. He then completed the remaining journey at 5.5 km/h. Who would arrive at the finishing line first?

37. Don and Andy have some marbles. If Don gives some marbles to Andy, then the number of marbles that Don has is twice what Andy has. If Andy gives the same number of marbles to Don, the number of marbles that Don has is 4 times what Andy has. How many marbles does each of them have at first?

38. In the figure below, AB = 20 cm, AD = 10 cm and the area of quadrilateral EFGH is 15 cm^2.
Find the total area of the shaded regions.

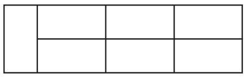


39. Alice, Bernard and Colin draw 3 cards each from nine cards numbered from 1 to 9.

Alice: The product of my numbers is 48.
Bernard: The sum of my numbers is 15.
Colin: The product of my numbers is 63.

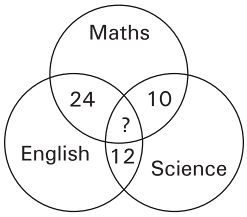
Find the three cards that each of them draws.

1. It suffices to consider four scenarios: 1 vertical, 3 vertical, 5 vertical and 7 vertical.
1 vertical: example
4 ways



3 vertical: 10 ways
5 vertical: 6 ways
7 vertical: 1 way
Ans: 21 ways

2.



28 + 41 + 20 = 89
24 + 12 + 10 = 46 students are counted twice.
89 – 46 = 43 students like either 1 or 2 subjects.
Number of students = 64
64 – 43 = **21 students** like all the three subjects.

3. It is sufficient to look only at the product of 1, 3, 5, 7 and 9.
1 × 3 × 5 × 7 × 9 = 945, where the digit in the ones place is 5.
Hence, we have 5 × 5 × … × 5.
The value of the digit in the ones place is **5**.

4. There are 7 pairs of 7 in 1 + 2 + … + 7 + 6 + … + 1.
Therefore,

$$\frac{1 + 2 + \ldots + 7 + 6 + \ldots + 1}{7777777 \times 7777777}$$
$$= \frac{7 \times 7}{7777777 \times 7777777}$$
$$= \frac{1}{1111111 \times 1111111}$$
$$= \mathbf{\frac{1}{1234567654321}}$$

5. **Scenario 1: Using line A as base**
$^5C_2 = \frac{5 \times 4}{1 \times 2}$
$= 10$
Total number of triangles, $10 \times {}^8C_1 = 80$.
Scenario 2: Using line B as base
$^8C_2 \times {}^5C_1 = \frac{8 \times 7}{1 \times 2} \times 5$
$= 28 \times 5$
$= 140$
Ans: 220 ways

6. Both of them will cover half of a circumference if they run towards each other. Jonathan needs to cover half a circumference in order to catch up with Cindy if they run in the same direction.
Let the speed of Jonathon and Cindy be J and C respectively.
We can write their speeds as follows:
To meet up:
(J + C) × 40 = half a circumference
To catch up:
(J – C) × 280 = half a circumference
Equating the two statements, we have
(J + C) × 40 = (J – C) × 280
J + C = (J – C) × 7
J + C = 7J – 7C
8C = 6J
J : C = 8 : 6
= 4 : 3
The ratio of Jonathan's speed to Cindy's speed is **4 : 3**.

7. Use the identity $a^2 - b^2 = (a - b)(a + b)$
$= (1 - 2)(1 + 2) + (3 - 4)(3 + 4) + \cdots + (2017 - 2018)(2017 + 2018)$
$= -(1 + 2 + 3 + 4 + \cdots + 2017 + 2018)$
$= -\frac{(1 + 2018) \times 2018}{2}$
$= \mathbf{-2\,037\,171}$

8. 5 × 7 = 35
2018 ÷ 5 = 403 R 3
2018 ÷ 7 = 288 R 2
2018 ÷ 35 = 57 R 23
Number of multiples of 5 or 7
= 403 + 288 – 57
= **634**

9. **Method 1:**
List down all the possible numbers in the table below.

Original number	New number	Difference
198	891	693
765	567	198
387	783	396

Therefore, the original number is **387**.

Method 2:
Let the 3-digit number be abc.
Hence, we have cba – abc = 396.
100c + 10b + a – 100a – 10b – c = 396
99c – 99a = 396
c – a = 396 ÷ 99 = 4
c = 4 + a ----- (1)
b – c = 1
c = b – 1 ----- (2)
Substitute (2) into (1):
b – a = 5
b = 5 + a ----- (3)
a + b + c = 18----- (4)
Substitute (1) and (3) into (4):
a + 5 + a + 4 + a = 18
3a + 9 = 18
3a = 9
a = 9 ÷ 3
= 3
Substitute a = 3 into (3): b = 8
Substitute a = 3 into (1): c = 7
The original number is **387**.